GAYATRI VIDYA PARISHAD COLLEGE OF ENGINEERING FOR WOMEN

(Autonomous)

(Affiliated to Andhra University, Visakhapatnam)

II B.Tech-I Semester Regular Examinations, November-2025

COMPLEX VARIABLES AND STATISTICAL METHODS

(EEE branch)

1. All questions carry equal marks
2. Must answer all parts of the question at one place

Time: 3 Hrs SCHEME OF EVALUATION Max. Marks: 70

UNIT-I

1. a.

$$\left.\begin{aligned}\frac{\partial u}{\partial x} &= e^x\big[(1+x)\cos y - y\sin y\big]\\ \frac{\partial u}{\partial y} &= e^x\big[-x\sin y - y\cos y - \sin y\big].\end{aligned}\right\} \text{— 2m}$$

Let v be the harmonic conjugate of u. Then

$$f'(z) = \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y} = e^x\big[(1+x)\cos y - y\sin y\big] + ie^x\big[x\sin y + y\cos y + \sin y\big]. \text{ — 1m}$$

By Milne-Thomson's method,

$$f'(z) = e^z\big[(1+z) - 0\big] + ie^z\big[0+0+0\big] = e^z(1+z). \text{ — 2m}$$

Integrating w.r.t. z, we get

$$f(z) = \int e^z(1+z)dz + c = ze^z + c \text{ — 2m}$$

1. b. Let $f(z) = u(x,y) + iv(x,y) = \sqrt{|xy|}$ so that $u(x,y) = \sqrt{|xy|}$ and $v(x,y) = 0$. — 1m

At the origin,

$$\frac{\partial u}{\partial x} = 0;\ \frac{\partial u}{\partial y} = 0;\ \frac{\partial v}{\partial x} = 0;\ \frac{\partial v}{\partial y} = 0 \text{ — 2m}$$

Hence, the Cauchy-Riemann equations are satisfied at the origin.

Now,

$$f'(0) = \lim_{z\to 0}\frac{f(z)-f(0)}{z-0} = \lim_{z\to 0}\frac{\sqrt{|xy|}-0}{x+iy} = \lim_{z\to 0}\frac{\sqrt{|xy|}}{x+iy}. \text{ — 2m}$$

If $z \to 0$ along the straight line $y = mx$, then $x \to 0, y \to 0$.

$$\therefore f'(0) = \lim_{x \to 0} \frac{\sqrt{|mx^2|}}{x + imx} = \lim_{x \to 0} \frac{x\sqrt{|m|}}{x(1+im)} = \lim_{x \to 0} \frac{\sqrt{|m|}}{1+im} = \frac{\sqrt{|m|}}{1+im}. \text{ — 2M}$$

Since the limit depends on m, for different paths, we have different limits. So the limit is not unique and hence $f'(0)$ does not exist. Thus $f(z)$ is not analytic at the origin, even though the C-R equations are satisfied at the origin.

2. a. C is the circle $|z - i| = 2$ with centre at $(0, 1)$ and a radius of 2.
The singular points are $z = -1, 2$. —— 1M
$z = -1$ lies inside C and $z = 2$ lies outside C. —— 1M
Let $f(z) = \dfrac{z-1}{z-2}$. Therefore, $f(z)$ is analytic inside and on C. — 1M
By Cauchy's integral formula, we have

$$\oint_C \frac{f(z)}{(z-a)^{n+1}} dz = \frac{2\pi i}{n!} f^{(n)}(a)$$

$$\Rightarrow \oint_C \frac{f(z)}{(z+1)^2} dz = \frac{2\pi i}{1!} f'(-1) = 2\pi i f'(-1) = -\frac{2\pi i}{9}. \text{ — 4M}$$

2. b. The integrand has singularities at $z = 1$ and $z = 2$. —— 1M
Both singular points lie inside C. —— 1M
Let $f(z) = \cos \pi z^2$.
$\therefore f(z)$ is analytic inside and on C. By Cauchy's integral formula, —— 1M

$$\oint_C \frac{f(z)}{(z-1)(z-2)} dz = \oint_C \frac{f(z)}{z-2} dz - \oint_C \frac{f(z)}{z-1} dz$$

$$\Rightarrow \oint_C \frac{\cos \pi z^2}{(z-1)(z-2)} dz = 2\pi i f(2) - 2\pi i f(1) = 4\pi i. \text{ — 4M}$$

UNIT-II

3. a. The poles of $f(z)$ are given by $z = 1, z = -\dfrac{3}{2}$. —— 1M
Both poles lie inside C. —— 1M

$$\therefore \text{Res}\{f(z); 1\} = \frac{1}{(2-1)!} \lim_{z \to 1} \frac{d}{dz}\left[(z-1)^2 f(z)\right] = 2 \text{ — 1M}$$

$$\text{Res}\left\{f(z); \frac{-3}{2}\right\} = \lim_{z \to \frac{-3}{2}} \left(z + \frac{3}{2}\right) f(z) = -2. \text{ — 1M}$$

Hence by Cauchy's residue theorem,

$$\oint_C f(z)dz = 2\pi i \times \big(\text{sum of the residues at the poles within } C\big)$$

$$\Rightarrow \oint_C \frac{12z-7}{(2z+3)(z-1)^2}dz = 2\pi i\big[2-2\big] = 0. \quad \text{—— 3m}$$

3. b. To evaluate the given integral, consider

$$\oint_C \frac{dz}{(z^2+a^2)^2}dz = \oint_C f(z)dz \quad \text{—— 1m}$$

where C is the contour consisting of the upper semi-circle C_R of radius R together with the part of the real axis from $-R$ to R.

$$\therefore \oint_C f(z)dz = \int_{C_R} f(z)dz + \int_{-R}^{R} f(x)dx. \quad \cdots\cdots \quad (1) \quad \text{—— 1m}$$

The poles of $f(z)$ are given by $(z^2+a^2)^2 = 0 \Rightarrow z = \pm ai$ (twice) which are poles of order 2.

But only $z = ai$ lies inside C.

$$\therefore \operatorname{Res}\{f(z); ia\} = \frac{1}{(2-1)!}\lim_{z\to ia}\frac{d}{dz}\Big\{(z-ia)^2 f(z)\Big\} = \frac{1}{4a^3 i} \quad \text{—— 1m}$$

Hence by Cauchy's residue theorem, we have

$$\oint_C f(z)dz = 2\pi i \times \big(\text{sum of the residues of } f(z) \text{ at poles within } C\big) = 2\pi i\Big(\frac{1}{4a^3 i}\Big) = \frac{\pi}{2a^3}. \quad \text{—— 1m}$$

Substituting in (1), we get

$$\int_{-R}^{R} f(x)dx + \int_{C_R} f(z)dz = \frac{\pi}{2a^3}. \quad \text{—— 2m}$$

But $\int_{C_R} f(z)dz \to 0$ as $z = Re^{i\theta}$ and $R \to \infty$.

Hence

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{\infty} \frac{1}{(x^2+a^2)^2}dx = \frac{\pi}{2a^3} \Rightarrow \int_0^{\infty} \frac{dx}{(x^2+a^2)^2} = \frac{\pi}{4a^3} \quad \text{—— 1m}$$

4. a.

$$f(z) = \frac{z^2-1}{(z+2)(z+3)} = 1 - \frac{5z+7}{(z+2)(z+3)} = 1 + \frac{3}{z+2} - \frac{8}{z+3}$$ — 2m

$f(z)$ is not analytic at $z=-2$ and $z=-3$.

$f(z)$ is analytic in the annular region $2<|z|<3$ about $z=0$.

When $2<|z|<3$. For $2<|z| \Rightarrow \frac{2}{|z|}<1$ and $|z|<3 \Rightarrow \left|\frac{z}{3}\right|<1$. Hence — 1m

$$\begin{aligned} f(z) &= 1 + \frac{3}{z\left(1+\frac{2}{z}\right)} - \frac{8}{3\left(1+\frac{z}{3}\right)} \\ &= 1 + \frac{3}{z}\left(1+\frac{2}{z}\right)^{-1} - \frac{8}{3}\left(1+\frac{z}{3}\right)^{-1} \\ &= 1 + \frac{3}{z}\sum_{n=0}^{\infty}(-1)^n\left(\frac{2}{z}\right)^n - \frac{8}{3}\sum_{n=0}^{\infty}(-1)^n\left(\frac{z}{3}\right)^n. \end{aligned}$$ 2m

4. b. Find the sum of residues of $f(z) = \frac{z^3}{(z-2)^2(z-3)}$ at its poles.

Solution: The poles of $f(z)$ are given $z=2, z=3$. — 1m

$z=2$ is a pole of order 2 and $z=3$ is a simple pole. — 1m

Residue of $f(z)$ at $z=3$ is $\text{Res}\{f(z);3\}=27$. — 2m

Residue of $f(z)$ at $z=2$ is $\text{Res}\{f(z);2\}=-20$. — 2m

Sum of the residues$=27-20=7$. — 1m

UNIT-III

5. a. E_1: The student is a girl, E_1: The student is a boy, M: The student is studied mathematics.

$$\therefore P(E_1) = 60\% = 0.6 \text{ and } P(E_2) = 40\% = 0.4$$ — 1m

Probability that mathematics is studied given that the student is a boy

$$P(M|E_2) = 0.25; \; P(M|E_1) = 0.1.$$ — 1m

(i) By theorem on total probability, probability that mathematics is studied

$$P(M) = P(E_1)P(M|E_1) + P(E_2)P(M|E_2) = 0.16$$ — 2m

(ii) By Baye's theorem, probability that a mathematics student is a girl

$$P(E_1|M) = \frac{P(E_1)P(M|E_1)}{P(M)} = 0.375.$$ — 3m

5. b. If the random variable X assigns the sum of its numbers in S.

$$X(s) = X(a,b) = a+b = \{2,3,4,5,6,7,8,9,10,11,12\}$$ — 1 m

$\therefore$ The discrete probability distribution is

$X = x_i$	2	3	4	5	6	7	8	9	10	11	12
$P(X = x_i)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

— 3m

$$\therefore \text{Mean } \mu = E(X) = \sum_i x_i p_i = 7.$$ — 2M

6. a. The probability of a man hitting a target $p = \dfrac{1}{3}$

The probability of a man not hitting a target $q = 1 - p = \dfrac{2}{3}$ — 1m

Number of trails$= n = 5$

Probability of hitting the target x times out of 5 times

$$P(X = x) = \binom{n}{x} p^x q^{n-x} = \binom{5}{x}\left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{5-x}, \quad x = 0, 1, \cdots, 5$$ — 1m

(i) If the man fires 5 times, probability of his hitting the target at least twice is given by

$$P(X \geq 2) = 1 - P(X < 2) = 0.5391.$$ — 2m

(ii) The number of times he should he fire, so that the probability of hitting the target at least once is greater than 90% is calculated as follows:

$$P(X \geq 1) > 90\% \Rightarrow P(X \geq 1) > 0.9 \Rightarrow \left(\frac{2}{3}\right)^n < 0.1$$

For $n = 6$, $\left(\dfrac{2}{3}\right)^6 = 0.088 < 0.1$. — 3m

Hence, the man must fire 6 times so that the probability of hitting the target at least once is more than 90%.

6. b. Let μ and σ be the mean and standard deviation of the distribution.

Then $\mu = 68$ and $\sigma = 3$.

Let X be the random variable which denotes the masses of a student. $Z = \dfrac{X - \mu}{\sigma}$ — 1m

(i)

$$\therefore P(X > 72) = P(Z > 1.33) = 0.0918.$$ — 2m

$\therefore$ Number of students with masses more than 72 kg$=NP(X > 72) = 300 \times 0.0918 = 27.54 \cong 28$ (approximately).

(ii)

$$\therefore P(X \leq 64) = P(Z \leq -1.33) = 0.0918. \quad \text{— 2m}$$

$\therefore$ Number of students with masses less than or equal to 64 kg$=NP(X \leq 64) = 300 \times 0.0918 = 27.54 \cong 28$ (approximately).

(iii)

$$\therefore P(65 \leq X \leq 71) = P(-1 \leq Z \leq 1) = 0.6826. \quad \text{— 2m}$$

$\therefore$ Number of students with masses between 65 kg 71 kg$=NP(65 \leq X \leq 71) = 300 \times 0.6826 = 204.78 \cong 205$. (approximately).

UNIT-IV

7. a. (i). Mean of the population,

$$\mu = \frac{2+3+6+8+11}{5} = \frac{30}{5} = 6. \quad \text{— 1m}$$

(ii). Variance of the population,

$$\sigma^2 = \sum_{i=1}^{5} \frac{(x_i-\mu)^2}{N} = \frac{16+9+0+4+25}{5} = 10.8 \Rightarrow \sigma = \sqrt{10.8} = 3.29 \quad \text{— 1m}$$

Sampling without replacement (finite population)

All possible samples of size two that can be drawn without replacement are

$$\left\{\begin{array}{cccc} (2,3) & (2,6) & (2,8) & (2,11) \\ & (3,6) & (3,8) & (3,11) \\ & & (6,8) & (6,11) \\ & & & (8,11) \end{array}\right\} \quad \text{— 1m}$$

The corresponding sample means are

$$\left\{\begin{array}{cccc} 2.5 & 4 & 5 & 6.5 \\ & 4.5 & 5.5 & 7 \\ & & 7 & 8.5 \\ & & & 9.5 \end{array}\right\} \quad \text{— 1m}$$

(iii). The mean of the sampling distribution of means

$$\mu_{\overline{X}} = \frac{2.5+4+5+6.5+4.5+5.5+7+7+8.5+9.5}{10} = \frac{60}{10} = 6. \quad \text{— 2M}$$

$\therefore$ Mean of the population=Mean of the sampling distribution of means i.e., $\mu_{\overline{X}} = \mu$.

(iv) The variance of sampling distribution of means

$$\sigma^2_{\overline{X}} = \frac{(2.5-6)^2+(4-6)^2+\cdots+(9.5-6)^2}{10} = \frac{40.5}{10} = 4.05 \quad \Big\} \text{2M}$$

$\therefore$ Standard deviation $\sigma_{\overline{X}} = \sqrt{4.05} = 2.01$.

7. b. We now find $P(510 \leq \overline{x} \leq 520)$.

We know that $z = \dfrac{\overline{x}-\mu}{\sigma/\sqrt{n}}$. —— 1M

If $\overline{x}_1 = 510$, $z_1 = \dfrac{\overline{x}_1-\mu}{\sigma/\sqrt{n}} = \dfrac{510-513.3}{31.5/\sqrt{40}} = -0.6$ —2M

If $\overline{x}_2 = 520$, $z_2 = \dfrac{\overline{x}_2-\mu}{\sigma/\sqrt{n}} = \dfrac{520-513.3}{31.5/\sqrt{40}} = 1.4$. —2M

$$\therefore P(510 \leq \overline{x} \leq 520) = P(z_1 \leq z \leq z_2) = 0.645 \quad \text{— 2M.}$$

8. a. Let μ be the mean of the population.

$\therefore$ The confidence interval is given by

$$\overline{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \overline{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}. \quad \text{—— 1M}$$

Now $\overline{x} - Z_{\alpha/2}\dfrac{\sigma}{\sqrt{n}} = 21.6 - \dfrac{1.96 \times 5.1}{\sqrt{100}} = 20.6$ —2M

and $\overline{x} + Z_{\alpha/2}\dfrac{\sigma}{\sqrt{n}} = 21.6 + \dfrac{1.96 \times 5.1}{\sqrt{100}} = 22.6$ —2M

Hence $(20.6, 22.6)$ is the confidence interval for the population mean μ.

8. b. The maximum error $E = Z_{\alpha/2}\left(\dfrac{\sigma}{\sqrt{n}}\right)$. —— 1M

Given that $n = 100$, S.D $\sigma = 5$.

Confidence limit $= 95\%$

$\Rightarrow Z_{\alpha/2} = 1.96$. —— 2M

$\therefore$ Maximum error $E = Z_{\alpha/2}\left(\dfrac{\sigma}{\sqrt{n}}\right) = (1.96)\left(\dfrac{5}{10}\right) = 0.98$. — 4M

UNIT-V

9. a. 1. Null hypothesis H_0: $\mu_1 = \mu_2$, i.e., there is no significant difference in heights of Australians and English men

2. Alternative hypothesis H_1: $\mu_1 < \mu_2$ (Left-tailed test)

3. Level of significance: $\alpha = 0.05$

2m

4. Test statistic:

$$Z = \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = -9.9.$$ 2m

i.e., calculated value of $|Z| = 9.9$.

5. Critical value: Since $\alpha = 0.05 \Rightarrow Z_\alpha = 1.645$ (for left-tailed test). 1m

6. Decision: Since calculated value of Z is greater than tabulated value of Z at 5% level of significance, the null hypothesis H_0 has to be rejected. 1m

Conclusion: Australians are on the average taller than English men. 1m

9. b.

Calculation for sample means and S

x	70	67	62	68	61	68	70	64	64	66
$x - \overline{x}$	4	1	−4	2	−5	2	4	−2	−2	0
$(x - \overline{x})^2$	16	1	16	4	25	4	16	4	4	0

2M

Sample mean $\overline{x} = \frac{\sum x}{n} = 66$ and $S^2 = \frac{1}{n-1}\sum(x - \overline{x})^2 = 10 \Rightarrow S = \sqrt{10} = 3.16$.

1. Null hypothesis H_0: The average height is not greater than 64 inches

i.e., $\mu = 64$ inches

2. Alternative hypothesis H_1: $\mu > 64$ (Right-tailed test)

3. Level of significance: $\alpha = 0.05$

2m

4. Test statistic:

$$t = \frac{\overline{x} - \mu}{S/\sqrt{n}} = 2.$$ 2m

i.e., calculated value of $|t| = 2$.

5. Critical value: Degrees of freedom $\nu = n - 1 = 10 - 1 = 9$.
The table value of t for 9 degrees of freedom at 5% level of significance for right-tailed test is 1.833. 1m

6. Decision: Since calculated value of t >tabulated value of t, the null hypothesis H_0

has to be rejected.

Conclusion: The average height is greater than 64 inches.

10. a. Here $n = 600$.

X=Number of smokers in city=325

p=Sample proportion of smokers in city=$\frac{X}{n} = \frac{325}{600} = 0.5417$

P=Population proportion of smokers in city=$\frac{1}{2} = 0.5$

$Q = 1 - P = 0.5$

1M

1. Null hypothesis H_0: The number of smokers and non-smokers are equal in the city i.e., $P = 0.5$
2. Alternative hypothesis H_1: $P > 0.5$ (Right-tailed test)
3. Level of significance: $\alpha = 0.05$

1M

4. Test statistic:

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = 2.04.$$

2M

i.e., calculated value of $|Z| = 2.04$.

5. Critical value: Since $\alpha = 0.05 \Rightarrow Z_\alpha = 1.645$ (for right-tailed test). 1M

6. Decision: Since calculated value of Z is greater than tabulated value of Z at 5% level of significance, the null hypothesis H_0 has to be rejected. 1M

Conclusion: Proportion of smokers in city is more than 50% and majority of men in the city are smokers. 1M

10. b. Given $n_1 = 5$ and $n_2 = 6$.

Now $\overline{x} = \frac{\sum x_i}{n_1} = \frac{41150}{5} = 8230$ and $\overline{y} = \frac{\sum y_i}{n_2} = \frac{47640}{6} = 7940$.

x_i	$x_i - \overline{x}$	$(x_i - \overline{x})^2$	y_i	$y_i - \overline{y}$	$(y_i - \overline{y})^2$
8260	30	900	7950	10	100
8130	-100	10000	7890	-50	2500
8350	120	14400	7900	-40	1600
8070	-160	25600	8140	200	40000
8340	110	12100	7920	-20	400
			7840	-100	10000
41150		63000	47640		54600

2M

Here $\sum(x_i - \overline{x})^2 = 63000$ and $\sum(y_i - \overline{y})^2 = 54600$.

$$\therefore S_1^2 = \frac{\sum(x_i - \overline{x})^2}{n_1 - 1} = \frac{63000}{4} = 15750 \text{ and } S_2^2 = \frac{\sum(y_i - \overline{y})^2}{n_2 - 1} = \frac{54600}{5} = 10920.$$

F-**test:**

1. Null hypothesis H_0: $\sigma_1^2 = \sigma_2^2$, i.e., the variances of success are equal
2. Alternative hypothesis H_1: $\sigma_1^2 \neq \sigma_2^2$ (two-tailed test)
3. Level of significance: $\alpha = 0.05$

2M

4. Test statistic:

$$F = \frac{S_1^2}{S_2^2} = \frac{15750}{10925} = 1.44 \qquad [\because S_1^2 > S_2^2]$$ — 1M

Therefore, calculated value of F=1.44

5. Critical value: Degrees of freedom $= (n_1 - 1, n_2 - 1) = (4, 5)$.

Tabulated value of F for $(4, 5)$ d.f at 5% level of significance is 5.19

6. Decision: Since calculated value of F is less than tabulated value of F, we accept the null hypothesis H_0.

1M

Conclusion: The variances of two samples are equal. — 1M

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